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FINAL REPORT

ANALYSIS OF DATA ON WATER WAVES

VOLUME I

THEORETICAL DEVELOPMENTS RELATED TO WATER WAVES GENERATED BY EXPLOSIONS IN DEEP WATER

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ANALYSIS OF DATA ON WATER WAVES

VOLUME I THEORETICAL DEVELOPMENTS RELATED TO WATER WAVES GENERATED BY EXPLOSIONS IN DEEP WATER

Prepared by:

R. W. Whalin

Prepared for:

United States Army Corps of Engineers
Waterways Experiment Station
Vicksburg, Mississippi

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Tetra Tech, Inc.
630 North Rosemead Boulevard
Pasadena, California 91107

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PREFACE

This study involved tasks related to three WES programs. The initial program, DASA Subtask 14.020 Deep Water Wave Study is presented in Volume I of this report. The latter programs DASA Subtasks 14.080 Mono Lake Field Experiment, and 14.096 Shallow Water Wave Study are covered in Volumes II and III respectively.

2. THEORETICAL DEVELOPMENTS RELATED TO WATER WAVES GENERATED BY EXPLOSIONS IN DEEP WATER

This section has been written with the objective of tracing the historical development of the multitude of papers which have contributed to the theory presently being applied for predicting water waves generated by deep water explosions in a constant depth environment. A general theoretical presentation is included and various special cases are presented with appropriate reference to original contributions. The mathematical development is carried out in detail to minimize the difficulties for readers not thoroughly familiar with the theory. A discussion of the initial conditions required for performing predictions is given along with an evaluation of the parameters using the experimental data resulting from this program. Finally, possible modifications, extensions, and applications of the theory are summarized in a detailed format which can be used as a guideline for further theoretical development.

2.1 Historical Theoretical Developments Related to Explosion Waves

The linear theory of water waves generated by explosions in deep water of constant depth consists of solving the Laplace equation in the appropriate coordinate system subject to the linearized free surface condition, the bottom condition and initial conditions, equivalent to the position and normal velocity or velocity potential of a fluid surface at one boundary. Several sets of initial conditions can be considered:

1. An initial deformation and velocity field on the free surface
2. An initial shape of the free surface and an applied pressure
3. An initial deformation and velocity field on the bottom.

The first theoretical contributions on the theory of waves produced by a local disturbance in deep water can be found in the classical memoirs of Cauchy (Ref. 1) and Poisson (Ref. 2). These works were performed independently and yield essentially equivalent results for two-dimensional problems. A mathematical analysis of both papers was performed by Burkhardt (Ref. 3), and Lamb (Ref. 4) has reviewed the theory and presented it in a consistent manner including contributions of his own. The two principle shortcomings of these early papers were 1) the approximations necessary for evaluating the integrals, and 2) confining the initial elevation or impulse to the vicinity of the origin. Lamb (Ref. 4) improved the mathematical approximations by observing that he could make use of a tabulation of the power series used by Rayleigh (Ref. 5) which appeared in the theory of Frenels diffraction integrals.

The mathematical difficulties which burdened the early works were removed by Kelvin (Ref. 6) when he developed the method of stationary phase for approximating the integral at large distances from the source of the disturbance. Many authors made contributions to the understanding of these early classical papers with the development of the concept of group and phase velocities, a description of the phenomena of waves in a dispersive medium, and various mathematical methods of approximation. A few of the more notable of these developments were due to Watson (Ref. 7), Burnside (Ref. 8), Dawson (Ref. 9), Terazawa (Ref. 10), Pidduck (Ref. 11) Thompson (Ref. 12), Pidduck (Ref. 13), Stokes (Ref. 14), and Havelock (Ref. 15).

The early classifical results above were supplemented by several papers in Japan on the waves produced by a bottom deformation, the impetus

for the consideration of this problem being the phenomena of tsunami waves produced by seismic disturbances of the sea bed. The earlier papers were by Hasegawa and Sano (Ref. 16), Sano (Ref. 17), and Terazawa (Ref. 18).

Further theoretical developments relating to explosion waves were practically non-existent, in England, Japan, and the U.S., until World War II when a need for more detailed knowledge about underwater explosion characteristics precipitated a rapid development of the theory. At this time, the work in Japan was still related to the tsunami problem, some of the more notable contributions being made by Nomura (Ref. 19), Segawa and Kanai (Ref. 20), Takahashi (Ref. 21, 22, 23), Ichiye (Ref. 24), Honda and Nakamura (Ref. 25), Nakamura (Ref. 26), Unoki and Nakano (Ref. 27, 28, 29), and Kajiura (Ref. 30). This latter paper by Kajiura is the most notable in that it presents the entire linear theory in a self-contained consistent manner which can be applied for sea bed disturbances, surface disturbances, pressure distributions, and an initial velocity distribution. A general approach is taken to the problem and it is solved by the use of a time dependent Green's function with several special cases being worked out for the asymptotic solution. This paper and related references will be referred to later during the development of the mathematical formulation and solution.

Extensive work was performed both in England and the United States during and immediately after World War II on what is commonly referred to as the wave "generation mechanism". This phrase means the early explosion behavior, i.e., the propagation of the detonation wave, production of explosive gases, formation of an underwater bubble, and bubble oscillations

and migration. Many of these papers are collected in Refs. 31 and 32 and much of the work also appears in Cole's well known book (Ref. 33). In addition, Snay (Refs. 34-36) has made notable contributions over the past 15 years to the understanding of explosion bubble phenomena. Bubble phenomena will not be discussed in this report due to the vast amount of literature on the subject and the fact that state of the art prediction techniques do not require any knowledge of bubble phenomena.

With the advent of nuclear explosions, a new and perhaps catastrophic problem was foreseen: the possible generation of a huge tsunami-like surface wave which might produce extensive coastal damage along the entire East, West or Gulf Coasts of the United States from a single detonation in deep water. This problem was attacked through a comprehensive program formulated and monitored through the Department of Defense. Although mathematical models had been developed, the applicability of these models was unknown as well as a realistic appraisal of the initial conditions. Stoker (Ref. 37) gave the Green's function for a three-dimensional disturbance in water of constant depth. The first successful attempt to formulate a realistic theory applicable to the prediction of explosion waves was published by Kranzer and Keller (Ref. 38). Prins (Ref. 39) performed a laboratory experiment in a two-dimensional tank (i.e, no radial spreading) where he elevated or depressed the water surface at one end of the tank and measured the subsequent wave profile at several stations for various ratios of water elevation to water depth. The initial major efforts of the DOD wave program consisted of a review of available theory and data, both HE and nuclear, and two experimental test series. The review was performed by Kaplan (Ref. 40) of Broadview Research Corporation (later

to become United Research Services). Many shortcomings were evident from this study, especially with respect to well-controlled experiments for obtaining data on both the wave generation mechanism (with the objective of relating initial conditions of the theoretical solution to the actual mode of generation) and the wave train produced by deep water explosions in water of constant depth. The experimental test series on the wave generation mechanism was performed at a small scale under reduced atmospheric pressure in a tank filled with oil. Hemispherical charges were detonated at the glass wall of the tank and high speed motion pictures were taken of the subsequent bubble and plume formation. The tank was not extensive enough for measurements of the wave profile to be taken at any significant distance from the charge. However, a very good qualitative understanding of the method of wave generation can be obtained from a careful observation of these films. This test series is reported on in a series of eight reports by Kaplan, et al. (Refs. 41-48). The other test series was performed by the Waterways Experiment Station and this volume constitutes the final report of that test series. Earlier reports of the data from the various field tests and an analysis of propagation characteristics may be found in Refs. 49-52. These field tests consisted of detonating 1/2 lb to 1000 lb charges of TNT and measuring the water waves at many stations in water of constant depth. Due to the comprehensive planning of these tests, the data has resulted in an evaluation of the initial conditions for the mathematical model as a function of yield and submergence depth resulting in predictions of a known reliability for HE charges over many orders of magnitude in charge weight. Extrapolations to nuclear explosions are still more uncertain; however, it is

generally accepted that periods can be predicted within 20% and amplitudes at least within a factor of 2 and probably considerably better. The uncertainty stems from a serious lack of deep water nuclear tests and the absence of reliable wave measurements from the few that were performed. The present state of the art prediction technique for deep water wave trains and the answer to associated questions relating to analytical approximations can be found in a series of reports by Whalin, et al, from Interstate Electronics Corporation (Ref. 53-59). A discussion of the applicability of the theoretical methods to areas near the source of disturbance is discussed by Whalin (Ref. 60). A relatively early exposition on impulsively generated water waves was authored by Van Dorn (Ref. 61) and some measurements from nuclear detonations are discussed in Ref. 62 and 63. Van Dorn also reported on a field program involving 14,400 lb HBX charges and the relationship between the measured profiles and those that could be reproduced through theoretical considerations (Ref. 64).

It should be noted that with the analysis of data from this test series and the availability of data from the 1965 and 1966 Mono Lake tests, the state of the art in predicting deep water explosion-generated waves is undergoing very rapid change and modification. A very interesting project which provided for an assessment of the state of the art became available through the prediction by Le Méhauté and Whalin (Ref. 65) of the waves to be produced from several explosions in Mono Lake. Preliminary comparisons with the subsequent experimental data indicate an approximate accuracy of 20% in amplitude and 10% in period. Another report by Whalin (Ref. 66) on this same project summarized the theoretical background for the predictions and indicated several modifications and extensions to the

theory. The following section of this report fully contains those findings of Ref. 66 and considerable supplementary information. An interesting application of the state of the art as of one year ago can be found in Refs. 67-69 where Le Méhauté, Whalin, et al. predicted coastal damage from waves near Los Angeles, Providence and New Orleans from offshore underwater and near shore air detonations.

The majority of research being conducted at this time is devoted to a better understanding of the modification of a wave train as it propagates over the continental slope and shelf, and significance of three-dimensional effects on run-up, the run-up from nonbreaking waves, shelf and harbor resonance, structural damage, and various other applications of the theory to specialized problems. The primary need for improving deep water predictions is a well documented nuclear test. Various modifications and extensions also should be carried out over the next several years as indicated in the following sections of this chapter.

2.2 Mathematical Formulation

The theoretical formulation of the problem will be presented in a general form as given by Kajiura (Ref. 30) and Whalin (Ref. 66); however, most details of the mathematical development are presented as well as a discussion of the contribution of various terms and the physical meaning of assumptions in both the original formulation and the method of solution. The energy of the source of disturbance also is given in detail.

A multitude of classical and recent theoretical papers was referenced in the preceding section. Regardless of their particular objectives, one facet is common to all of these papers, namely the use of a linear theory for surface waves in an inviscid and incompressible fluid. The assumption of constant depth is almost a necessity for obtaining closed form analytical solutions and also is common to the majority of publications related to this problem. The principle difference in these references consists of various assumptions for initial conditions, the use of two- or three-dimensional coordinate systems, and different methods of approximation for obtaining an analytical solution. The initial formulation and solution will include all previous publications as special cases and the solutions for special cases will be discussed in detail.

Our assumptions will consist of the following:

1. The water is incompressible
2. The fluid is non-viscous
3. The flow is irrotational
4. The fluid domain is of constant depth, d, and infinite in extent
5. The first order of approximation is assumed for the kinematic and dynamic conditions at the free surface.

The origin of the Cartesian coordinate system (x', y', z') is at the undisturbed free surface with the vertical axis z' being taken positive upwards. All variables are transformed to dimensionless form ($x = x'/d$, $y = y'/d$, $z = z'/d$, $t = t'\sqrt{g/d}$, $\eta = \eta'/d$, $\varphi = \varphi'/d\sqrt{gd}$, $p = (p'/\rho)/gd$, etc).

The kinematic and dynamic conditions at the free surface are

$$\varphi_x \eta_x - \varphi_z + \varphi_y \eta_y + \eta_t = 0$$

$$\eta + \varphi_t + \frac{1}{2}(\varphi_x^2 + \varphi_y^2 + \varphi_z^2) + p = 0$$

With assumption (5) on the previous page, these conditions are linearized and become

$$\varphi_z = \eta_t, \quad z = 0$$

$$\varphi_t = -\eta - p, \quad z = 0$$

The bottom condition is

$$\varphi_n' = w_B', \quad z = -h(x, y)$$

which, from assumption (4), becomes

$$\varphi_z' = w_B', \quad z' = -d$$

or in dimensionless form

$$\varphi_z = w_B, \quad z = -1$$

where w_B' is the assumed bottom velocity corresponding to a bottom deformation and φ' is the velocity potential, which, from assumptions

(1), (2), (3) and (4) assures the existence of a single valued function $\varphi(x, y, z, t)$ from which the velocity field can be derived by taking the gradient

$$\vec{v} = \operatorname{grad} \varphi$$

Since the velocity components are given by the above equation, it follows directly from the continuity equation

$$\operatorname{div} \vec{v} = 0$$

that the velocity potential, φ, is a solution of the Laplace equation

$$\nabla^2 \varphi \equiv \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$

and therefore φ is a harmonic function.

It should be noted that linearization places certain restrictions on the surface and bottom deformations that may be assumed. Specifically, both should be small compared with the wavelength λ' and water depth d. The condition $z'\lambda'^2/d^3 \lesssim 1$ should also be satisfied. However, we will see in the next sections that for practical purposes of prediction, the deformation may be much larger than one would normally expect for the theory to be valid.

This problem will be approached through the use of a time dependent Green's function, G, which is a solution of the Laplace equation

$$\nabla^2 G = 0, \qquad 0 > z > -1, \quad t \geq \tau$$

satisfying the free surface condition

$$G_{tt} + G_z = 0, \qquad z = 0$$

and the bottom condition

$$G_z = 0, \qquad z = -1$$

The mathematical conditions on the Green's function which must be satisfied to assure the existence of a unique solution are that G, G_x, G_y, G_z, G_t, G_{tx}, G_{ty}, and G_{tz} must be uniformly bounded for every t at $x, y \longrightarrow \infty$ and $(G - 1/R)$ must be bounded at any source point (x_o, y_o, z_o) where

$$R^2 = (x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2$$

At time $t = \tau$ the Green's function is assumed to satisfy the following initial conditions

$$G = G_t = 0, \qquad z = 0$$

The above equation and conditions are sufficient to determine the Green's function uniquely. For a derivation of the Green's function, in the case of three-dimensional motion in water of finite depth, the reader is referred to Stoker (Ref. 37). From Stoker

$$G(x_o, y_o, z_o; \tau \,|\, x, y, z; t) = \int_0^\infty \frac{J_o(\sigma \bar{r})}{\cosh \sigma} \Big[\sinh \sigma \left\{1 - |z - z_o|\right\} - \sinh \sigma \left\{1 + (z + z_o)\right\}$$

$$+ \frac{2}{\gamma^2} \left\{1 - \cos \gamma (t - \tau)\right\} \frac{\sigma}{\cosh \sigma} \cosh \sigma (1 + z) \cosh \sigma (1 + z_o) \Big] d\sigma, \quad 0 > z, \; z_o > -1$$

where

$$\bar{r}^2 = (x - x_o)^2 + (y - y_o)^2$$

$$\gamma^2 = \sigma \tanh \sigma$$

Upon making use of Green's formula, the above Green's function,, and symmetry properties of G, one obtains an expression for $\varphi_\tau (x, y, z, t)$. The integration of φ_τ from 0 to τ and subsequent substitution of φ into the boundary conditions gives the following expression for the free surface elevation, η

$$\eta + p = \frac{1}{4\pi} \iint_S \Big[\Big\{ (G_t \varphi_{z_o} - G_{\tau t} \eta)_{\tau=0} + \Big[\int_o^t p\, G_{\tau\tau t}\, d\tau + (p\, G_{\tau\tau})_{\tau=t} \Big] \Big\}_{z=z_o=0}$$

$$+ \Big[\int_o^t G_t\, \varphi_{z_o \tau}\, d\tau \Big]_{z_o=-1,\ z=0} \Big] dS$$

The details of the above derivation may be found in either Stoker (Ref. 37), Kajiura (Ref. 30) or Whalin (Ref. 66). The above expression for η represents the classical solution derived in a considerably more general form.

Lets now analyze the contribution of each term in the above equation

1. The first term represents the contribution of the initial velocity distribution, φ_{z_o}, assumed on the free surface.
2. The second term represents the contribution of the initial deformation, $\eta|_{\tau=0}$, of the free surface.
3. The third and fourth terms represent the contribution of the applied pressure, p.
4. The latter term represents the contribution of the bottom deformation, w_B being the vertical velocity of the bottom deformation.

2.3 Asymptotic Solution for Special Cases

The difficulties involved with the solution of the above equation depend upon the area where one wishes to obtain an estimate of the wave profile and the functional dependence of the initial conditions on the space and time coordinates. The solution can be integrated numerically regardless of the complexity of the initial conditions. However, in the case of waves generated by a single explosion, the problem is axially symmetric. This results in the space integration becoming the zero order Hankel transform of the initial mode of disturbance or the integral of the disturbance from 0 to t. If one cannot perform the Hankel transform analytically, the approximate integration method developed by Whalin (Ref. 60) may be used to accurately compute the amplitude time history. A closed form solution may be obtained by use of the method of stationary phase to approximate the integral over the wave number. The following examples illustrate the closed form solution for several cases of the various types of initial conditions.

Upon computation of the appropriate partial derivatives and expressing the solution in cylindrical coordinates, one obtains the following relationships.

$$G_t\big|_{z=0,\ z_o=0,\ \tau=0} = \int_0^\infty \frac{2\sigma}{\gamma} J_o(\sigma\bar{r}) \sin\gamma t \, d\sigma$$

$$G_{t\tau}\big|_{z=0,\ z_o=0,\ \tau=0} = \int_0^\infty 2\sigma J_o(\sigma\bar{r}) \cos\gamma t \, d\sigma$$

$$G_{\tau\tau t}\big|_{z=0,\ z_o=0} = -\int_0^\infty 2\ \sigma\ \gamma\ J_o\ (\sigma\ \bar{r})\ \sin\gamma\ (t-\tau)\ d\ \sigma$$

$$G_{\tau\tau}\big|_{z=0,\ z_o=0,\ \tau=t} = \int_0^\infty 2\ \sigma .\ J_o\ (\sigma\ \bar{r})\ d\ \sigma$$

$$G_{t}\big|_{z_o=-1,\ z=0} = \int_0^\infty \frac{2\ \sigma}{\gamma\ \cosh\ \sigma}\ J_o\ (\sigma\ \bar{r})\ \sin\gamma\ (t-\tau)\ d\sigma$$

$$J_o\ (\sigma\ \bar{r}) = J_o\ (\sigma\ r)\ J_o\ (\sigma\ r_o) + 2\sum_{n=1}^{\infty} J_n\ (\sigma\ r)\ J_n\ (\sigma\ r_o)\ \cos n\ (\theta-\theta_o)$$

Note that upon integration with respect to the source variable θ_o (assuming axial symmetry for all initial disturbances)

$$\int_o^{2\pi} J_o\ (\sigma\bar{r})\ d\ \theta_o = 2\pi\ J_o\ (\sigma\ r)\ J_o\ (\sigma\ r_o)$$

$$\eta + p = \int_o^\infty \Big[\int_o^\infty \frac{\sigma}{\gamma}\ J_o\ (\sigma\ r)\ J_o\ (\sigma\ r_o)\ \sin\gamma\ t\ d\sigma\ .\ \varphi_{z_o}\big|_{z_o=0,\ \tau=0}$$

$$+ \int_o^t J_o\ (\sigma\ r)\ J_o\ (\sigma\ r_o)\ \cos\gamma\ t\ d\ \sigma\ \ \eta\ (r_o)\big|_{\tau=0}$$

$$+ \int_o^t p\ (r_o,\ \tau) \int_o^\infty -\ \sigma\ \gamma\ J_o\ (\sigma\ r)\ J_o\ (\sigma\ r_o)\ \sin\gamma\ (t-\tau)\ d\ \sigma\ d\tau$$

$$+ \left(p\ (r_o,\ \tau) \int_o^\infty \sigma\cdot J_o\ (\sigma\ r)\ J_o\ (\sigma\ r_o)\ d\ \sigma\right)_{\tau=t}$$

$$+ \int_o^t \int_o^\infty \frac{\sigma}{\gamma\ \cosh}\ \sin\gamma\ (t-\tau)\ d\ \sigma\ \ \varphi_{z_o\ \tau}\big|_{z_o=-1}\ d\ \tau\Big]\ r_o\ d\ r_o$$

Note: If the initial surface deformation is defined in general by $\eta_o\ (r_o,\ \tau)$ then

$$\eta_o\ (r_o,\ \tau)\big|_{\tau=0} = \eta_o\ (r_o)$$

$$\varphi_{z_o}\big|_{z_o=0,\ \tau=0} = \frac{\partial}{\partial\tau}\eta_o(r_o,\tau)\big|_{\tau=0} = \eta_{o_\tau}(r_o, 0)$$

Now consider the following special cases:

Case 1: The applied pressure at the surface is impulsive at $\tau = 0^+$:

$$p(r_o, \tau) = p(r_o)\,\delta(0^+)$$

therefore

$$\int_0^t p(r_o, \tau)\, G_{\tau\tau t}\, d\tau + p(r_o, \tau)_{\tau=t} = I_o(r_o)\, G_{\tau\tau t}$$

All other terms are identically zero of the equation since $\varphi_{z_o} = 0$, $\eta(r_o) = 0$, $w_B = 0$. Therefore

$$\eta(r, t) = \int_0^\infty \sigma\,\gamma\, J_o(\sigma r)\sin\gamma t \int_0^\infty I_o(r_o)\, J_o(\sigma r_o)\, r_o\, dr_o\, d\sigma$$

Upon inspection one notices that

$$\int_0^\infty I_o(r_o)\, J_o(\sigma r_o)\, r_o\, dr_o = \overline{I_o}(\sigma) = \text{zero order Hankel transform}$$

of the initial impulse.

Therefore

$$\eta(r, t) = \int_0^\infty \sigma\,\gamma\,\overline{I_o}(\sigma)\, J_o(\sigma r)\sin\gamma t\, d\sigma$$

The above integral can be evaluated by the method of stationary phase subject to the following assumptions

1) The frequency of oscillation of $I_o(\sigma)$ is low compared with that of $J_o(\sigma r)$ and $\sin \gamma t$.

2) $J_o(\sigma r)$ may be approximated by the first term of its asymptotic expansion for large argument. Note: Additional terms in the asymptotic expansion will lead to a slightly better approximation; however, the increase in accuracy is small compared with the uncertainty in choosing the initial conditions.

We now proceed with the evaluation of the integral, preceded by the following theorem.

Theorem: If an integral is of the form

$$\int_A^B \chi(z)\, e^{\tau f(z)}\, dz$$

where $\chi(z)$ is of bounded variation, τ is a large positive constant, $\tau f(z)$ is analytic in (A, B), and $f(z) = \varphi + i\psi$, where $\varphi = 0$ and $\psi' = 0$ has a unique solution x_o in (A, B), then

$$\int_A^B \chi(z)\, e^{\tau f(z)}\, dz \approx \chi(x_o) \sqrt{\frac{2\pi}{\tau \psi''(x_o)}}\; e^{i\left\{\tau \psi(x_o) + \frac{\pi}{4} \operatorname{sqn} \psi'' x_o\right\}} + \theta\left(\frac{1}{\tau}\right)$$

Proceeding with the evaluation, we make the following assumptions

1) $\sigma\gamma \bar{I}_o(\sigma)$ has a low frequency of oscillation relative to $J_o(\sigma r)$ and $\sin \gamma t$.

2) $J_o(\sigma r)$ is approximated by the first term in the asymptotic expansion for large argument:

$$J_o(\sigma r) \approx \sqrt{\frac{2}{\pi \sigma r}} \cos\left(\sigma r - \frac{\pi}{4}\right) = \sqrt{\frac{2}{\pi\sigma r}}\; \frac{e^{i(\sigma r - \frac{\pi}{4})} + e^{-i(\sigma r - \frac{\pi}{4})}}{2}$$

Upon substitution one obtains

$$\eta(r, t) \approx \int_0^\infty \sqrt{\frac{2}{\pi\sigma r}}\; \sigma\gamma \bar{I}_o(\sigma) \left\{\frac{e^{i(\sigma r + \gamma t - \frac{\pi}{4})} - e^{i(\sigma r - \gamma t - \frac{\pi}{4})}}{4i} + \frac{e^{-i(\sigma r - \gamma t - \frac{\pi}{4})} - e^{-i(\sigma r + \gamma t - \frac{\pi}{4})}}{4i}\right\}$$

The expression for $\eta(r, t)$ is now in a form which can be evaluated by the method of stationary phase where $\varphi = 0$, $\psi = \pm(\sigma r \pm \gamma t - \pi/4)$ and the stationary phase points σ_o are solutions of $\psi'(\sigma) = 0$, where

$$\psi'(\sigma) = \pm r \pm \frac{\tanh \sigma + \sigma \operatorname{sech}^2 \sigma}{\sqrt{\sigma \tanh \sigma}} t$$

$$\psi''(\sigma) = \left(\frac{1}{4} \frac{[\tanh \sigma + \sigma \operatorname{sech}^2 \sigma]^2}{(\sigma \tanh \sigma)^{3/2}} + \frac{\operatorname{sech}^2 \sigma - \sigma \operatorname{sech}^2 \sigma . \tanh \sigma .}{(\sigma \tanh \sigma .)^{1/2}}\right) t$$

The stationary phase points are defined by

$$\varphi(\sigma) \equiv \frac{1}{2}\left\{\sqrt{\frac{\tanh \sigma}{\sigma}} + \frac{1}{\cosh^2 \sigma} \sqrt{\frac{\sigma}{\tanh \sigma}}\right\} = \frac{r}{t}$$

Only the second and third integrals have real stationary phase points. In the second integral

$$\psi(\sigma) = \sigma r - \gamma t - \frac{\pi}{4}$$

$$\psi'(\sigma) = r - \frac{\tanh \sigma + \sigma \operatorname{sech}^2 \sigma}{2\sqrt{\sigma \tanh \sigma}}$$

$$\psi''(\sigma) = -\left\{-\frac{1}{4} \frac{[\tanh \sigma + \sigma \operatorname{sech}^2 \sigma]^2}{[\sigma \tanh \sigma]^{3/2}} + \frac{\operatorname{sech}^2 \sigma - \sigma . \operatorname{sech}^2 \sigma \tanh \sigma}{[\sigma \tanh \sigma]^{1/2}}\right\}$$

The stationary phase points are defined by

$$\varphi(\sigma) \equiv \frac{1}{2}\left\{\sqrt{\frac{\tanh \sigma}{2}} + \frac{1}{\cosh^2 \sigma} \sqrt{\frac{1}{\tanh \sigma}}\right\} = \frac{r}{t}$$

and $\varphi''(\sigma)$ is positive.

In the third integral

$$\psi(\sigma) = -(\sigma r - \gamma t - \frac{\pi}{4})$$

$$\psi'(\sigma) = -\left\{ r - \frac{\tanh\sigma + \sigma\,\mathrm{sech}^2\sigma}{2\sqrt{\sigma\tanh\sigma}}\, t \right\}$$

$$\psi''(\sigma) = \left\{ -\frac{1}{4}\frac{[\tanh\sigma + \sigma\,\mathrm{sech}^2\sigma]^2}{[\sigma\tanh\sigma]^{3/2}} + \frac{\mathrm{sech}^2\sigma - \sigma\,\mathrm{sech}^2\sigma\tanh\sigma}{[\sigma\tanh\sigma]^{1/2}} \right\} t \quad .$$

The stationary phase points are defined by the same functional relationship as in the previous case, however $\psi''(\sigma)$ is negative in this case.

Upon application of the previous theorem

$$\eta(r,t) = \sigma\gamma\bar{I}_o(\sigma)\sqrt{\frac{2}{\pi\sigma r}}\sqrt{\frac{2\pi}{-\varphi'(\sigma)t}}\;\frac{e^{i\{(\sigma r - \gamma t - \frac{\pi}{4} + \frac{\pi}{4}(+1)\}} + e^{-i\{\sigma r - \gamma t - \frac{\pi}{4} - \frac{\pi}{4}(-1)\}}}{4i}$$

where $|\psi''(\sigma_o)|$ has been replaced by $(-\varphi'(\sigma)\,t)$. Upon substitution of $t = r/\varphi(\sigma)$ and simplification, one obtains

$$\eta(r,t) = \frac{2\bar{I}_o(\sigma)\sigma\gamma}{r}\sqrt{\frac{\varphi(\sigma)}{-\sigma\,\varphi'(\sigma)}}\left\{-\frac{e^{i(\sigma r - \gamma t)} + e^{-i(\sigma r - \gamma t)}}{4i}\right\}$$

$$= \frac{2\bar{I}_o(\sigma)\,\sigma\,\gamma}{r}\sqrt{\frac{\varphi(\sigma)}{-\sigma\,\varphi'(\sigma)}}\left[-\frac{1}{2}\sin(\sigma r - \gamma t)\right]$$

$$= -\frac{\bar{I}_o(\sigma)\,\sigma}{r}\sqrt{\frac{\varphi(\sigma)\tanh\sigma}{-\varphi'(\sigma)}}\,\sin(\sigma r - \sqrt{\sigma\tanh\sigma}\;t)$$

The above equation reduces to Kranzer and Keller's result when put in the form

$$\eta(r, t) = \frac{\overline{I}_o(\sigma)\,\sigma}{r} \sqrt{\frac{\varphi(\sigma)\tanh\sigma}{-\varphi'(\sigma)}} \sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$$

where

$$\varphi(\sigma) \equiv \frac{1}{2}\left\{\sqrt{\frac{\tanh\sigma}{\sigma}} + \frac{1}{\cosh^2\sigma}\sqrt{\frac{\sigma}{\tanh\sigma}}\right\} = \frac{r}{t}$$

$$\overline{I}_o(\sigma) = \int_0^\infty I_o(r_o)\, J_o(\sigma r_o)\, r_o\, d r_o$$

$$\lambda = \frac{2\pi}{\sigma}$$

$$T = \frac{2\pi}{\sqrt{\sigma\tanh\sigma}}$$

All variables above are in dimensionless form as previously noted.

Case 2: The applied pressure is a function of time from 0 to τ^* and zero thereafter

$$p = \begin{cases} p(r_o, \tau), & \tau \le \tau^* \\ 0, & \tau > \tau^* \end{cases}$$

therefore

$$\int_0^t p(r_o, \tau)\, G_{\tau\tau t}\, d\tau + [p(r_o, \tau)]_{\tau = t} = \int_0^{\tau^*} p(r_o, \tau)\, G_{\tau\tau t}\, d\tau$$

$$= \int_0^\infty -\sigma\gamma J_o(\sigma r)\, J_o(\sigma r_o) \int_0^{\tau^*} p(r_o, \tau) \sin\gamma(t - \tau)\, d\tau\, d\sigma$$

Let

$$\int_0^{\tau^*} p\,(r_o, \tau) \sin \gamma\,(t - \tau)\, d\tau = P\,(r_o, t)$$

then

$$\eta(r, t) = -\int_0^{\infty} \sigma \gamma J_o\,(\sigma r) \int_0^{\infty} P\,(r_o, t)\, J_o\,(\sigma r_o)\, r_o\, dr_o\, d\sigma$$

Let

$$\int_0^{\infty} P\,(r_o, t)\, J_o\,(\sigma r_o)\, r_o\, dr_o = \overline{P}\,(\sigma, t) = \text{zero order Hankel transform of } P(r_o, t)$$

$$\eta\,(r, t) = -\int_0^{\infty} \sigma\gamma\, \overline{P}(\sigma, t)\, J_o\,(\sigma r)\, d\sigma$$

The above integral also may be integrated by the method of stationary phase once $\overline{P}(\sigma, t)$ is known and provided that stationary phase points exist. Otherwise, if one cannot integrate directly (which is unlikely) then another approximate or numerical integration technique must be used.

<u>Case 3:</u> The applied pressure is non-zero for all time t

$$p = p\,(r_o, \tau)$$

therefore

$$\eta(r, t) = \int_0^{\infty} \left\{ \int_0^{t} p\,(r_o, \tau)\, G_{\tau\tau t}\, d\tau + \Big[p\,(r_o, \tau)\, G_{\tau\tau} \Big]_{\tau = t} \right\} r_o\, dr_o - P(r, t)$$

$$= \int_0^{\infty} \sigma \gamma J_o\,(\sigma r) \left[-\int_0^{\infty} P\,(r_o, t)\, J_o\,(\sigma r_o)\, r_o\, dr_o + \frac{1}{\gamma} \int_0^{\infty} p\,(r_o, t)\, J_o\,(\sigma r_o)\, r_o\, dr_o \right] d\sigma$$

$$- P(r, t)$$

where

$$P(r_o, t) = \int_0^t p(r_o, \tau) \sin \gamma (t - \tau) \, d\tau$$

Let

$$\overline{P}(\sigma, t) = \int_0^\infty P(r_o, t) J_o(\sigma r_o) r_o \, dr_o$$

$$\overline{p}(\sigma, t) = \int_0^\infty p(r_o, t) J_o(\sigma r_o) r_o \, dr_o$$

then

$$\eta(r, t) = \int_0^\infty \sigma \gamma J_o(\sigma r) \left\{ -\overline{P}(\sigma, t) + \frac{p(\sigma, t)}{\gamma} \right\} d\sigma - p(r, t)$$

Similar comments are applicable as in the previous case with regard to the integration.

Case 4: There is an initial surface deformation which is stationary at time zero. Therefore

$$\eta(r, t) = \int_0^\infty \int_0^\infty \sigma J_o(\sigma r) J_o(\sigma r_o) \cos \gamma t \, \eta_o(r_o) \, d\sigma \, r_o \, dr_o$$

$$= \int_0^\infty \sigma J_o(\sigma r) \cos \gamma t \int_0^\infty \eta_o(r_o) J_o(\sigma r_o) r_o \, dr_o \, d\sigma$$

$$= \int_0^\infty \overline{\eta}_o(\sigma) \sigma J_o(\sigma r) \cos \gamma t \, d\sigma$$

Upon evaluation by the method of stationary phase

$$\eta(r,t) \approx \frac{\bar{\eta}_o(\sigma)}{r} \sqrt{\frac{\sigma\,\varphi(\sigma)}{-\varphi'(\sigma)}} \cos\left(\sigma r - \sqrt{\sigma \tanh \sigma}\; t\right)$$

Special note should be taken of this case since it will form the basis of our prediction technique.

Case 5: An initial velocity field at time $t = 0$ with no surface deformation. Therefore

$$\eta(r,t) = \int_0^\infty \int_0^\infty \frac{\sigma}{\gamma} J_o(\sigma r) J_o(\sigma r_o) \sin \gamma t \left[\varphi_{z_o}\right]_{\substack{z_o=0 \\ \tau=0}} d\sigma\, r_o\, dr_o$$

$$= \int_0^\infty \frac{\sigma J_o(\sigma r) \sin \gamma t}{\gamma} \int_0^\infty [\varphi_{z_o}] J_o(\sigma r_o)\, r_o\, dr_o\, d\sigma$$

$$= \int_0^\infty \bar{\varphi}_{z_o}(\sigma) \frac{\sigma J_o(\sigma r)}{\gamma} \sin \gamma t\, d\sigma$$

$$\approx \frac{\bar{\varphi}_{z_o}(\sigma)}{r\sqrt{\tanh \sigma}} \sqrt{\frac{\varphi(\sigma)}{-\varphi'(\sigma)}} \sin\left(\sigma r - \sqrt{\sigma \tanh \sigma}\; t\right)$$

Case 6: An initial surface deformation and an initial velocity field.

The solution in this case is merely the sum of the solutions for Cases 4 and 5.

$$\eta(r,t) \approx \frac{1}{r}\sqrt{\frac{\varphi(\sigma)}{-\varphi'(\sigma)}} \left\{ \sqrt{\sigma}\, \eta_o(\sigma) \cos\left(\sigma r - \sqrt{\sigma \tanh \sigma}\; t\right) + \frac{\bar{\varphi}_{z_o}(\sigma)}{\sqrt{\tanh \sigma}} \sin\left(\sigma r - \sqrt{\sigma \tanh \sigma}\; t\right) \right.$$

Case 7: A bottom deformation of constant velocity w_B for a finite time τ^*. Therefore

$$(G_t \varphi_{z_o})_{\tau=0} = w_B \int_0^\infty \frac{2\sigma \sin \gamma t \, d\sigma}{\gamma \cosh \sigma}$$

$$\int_0^{\tau^*} G_{t\tau} \varphi_{z_o} \, d\tau = w_B \int_0^{\tau^*} \int_0^\infty \frac{J_o(\sigma \bar{r})}{\cosh \sigma} \, 2\sigma [-\cos \gamma(t-\tau)] \, d\sigma .$$

$$= - w_B \int_0^\infty \frac{2\sigma J_o(\sigma \bar{r})}{\cosh \sigma} \int_0^{\tau^*} \cos \gamma(t-\tau) \, d\tau \, d\sigma$$

$$= -2 w_B \int_0^\infty \frac{\sigma J_o(\sigma \bar{r})}{\cosh \sigma} \left[\frac{-\sin \gamma(t-\tau)}{\gamma} \right]_0^{\tau^*} d\sigma$$

$$= 2 w_B \int_0^\infty \frac{\sigma J_o(\sigma \bar{r})}{\gamma \cosh \sigma} [\sin \gamma(t-\tau^*) - \sin \gamma t] \, d\sigma$$

Therefore

$$\eta(r,t) = 2 w_B \int_0^\infty \frac{\sigma J_o(\sigma r)}{\gamma \cosh \sigma} [\sin \gamma(t-\tau^*) - \sin \gamma t] \int_0^\infty J_o(\sigma r_o) r_o \, dr_o \, d\sigma$$

$$= 2 w_B \int_0^\infty \frac{\bar{\eta}(\sigma) \, \sigma J_o(\sigma r)}{\gamma \cosh \sigma} [\sin \gamma(t-\tau^*) - \sin \gamma t] \, d\sigma$$

2.4 Evaluation of the Wave Amplitude for Areas Near the Source and Near the Front of the Wave Train

It has been demonstrated in a paper by Whalin (Ref. 54, 60) that the asymptotic solution obtained through integration by the method of stationary phase is valid within 5% provided the distance r where the amplitude is computed is approximately greater than 5 times the radius of the initial surface deformation. If one desires to predict the wave train closer to the source of disturbance, an alternate integration technique is necessary. The method described in this section consists of fitting a fourth degree polynomial to each half oscillation of the integrand and to then perform the integration directly.

The solution can be represented by

$$\eta(r, t) = \int_0^\infty f(\sigma)\, d\sigma$$

$$= \int_0^a f(\sigma)\, d\sigma + \int_a^\beta f(\sigma)\, d\sigma + \int_\beta^\infty f(\sigma)\, d\sigma$$

where a = a small number such that the oscillatory function of the integrand may be approximated by asymptotic expansion near $\sigma = 0$

β = a large value of σ such that the latter integral is negligible and is a zero of the integrand

$f(\sigma)$ is an oscillatory function of σ depending upon the nature of the initial disturbance. Having chosen the proper limits a and β, the first integral is integrated directly and the latter is negligible.

Let σ_i be the ordered zeros of the function $f(\sigma)$ in the interval $a < \sigma_i \leq \beta$, $i = 1, 2, \ldots, N$ and $\sigma_N = \beta$, then

$$f(\sigma) = K\left\{A_i'\,\sigma^4 + B_i'\,\sigma_3 + C_i'\,\sigma_2 + D_i'\,\sigma + E_i'\right\}$$

$$a \le \sigma \le \sigma_i, \quad i = 0$$

$$\sigma_{i-1} < \sigma \le \sigma_i, \quad i = 2, 3, \ldots, N$$

where K is a constant depending on the initial conditions

Upon substitution of the above expression for $f(\sigma)$ and subsequent term by term integration, the wave amplitude becomes

$$\eta(r,t) \approx K\left\{\frac{A_1'}{9}(\sigma_1^{9/2} - a^{9/2}) + B_1'(\sigma_1^{7/2} - a^{7/2}) + C_1'(\sigma_1^{5/2} - a^{5/2})\right.$$

$$+ \frac{D_1'}{3}(\sigma_1^{3/2} - a^{3/2}) + E_1'(\sigma_1^{1/2} - a^{1/2})$$

$$+ \sum_{i=2}^{i=N}\left[\frac{A_1'}{9}(\sigma_i^{9/2} - \sigma_{i-1}^{9/2}) + \frac{B_1'}{7}(\sigma_i^{7/2} - \sigma_{i-1}^{7/2}) + \frac{C_1'}{5}(\sigma_i^{5/2} - \sigma_{i-1}^{5/2})\right.$$

$$\left.\left. + \frac{D_i'}{3}(\sigma_i^{3/2} - \sigma_{i-1}^{3/2}) + E_i'(\sigma_i^{1/2} - \sigma_{i-1}^{1/2})\right]\right\}$$

where

$$A_i' = A_i$$

$$B_i' = (\sigma_i + \sigma_{i-1})\,A_i + B_i$$

$$C_i' = \sigma_i\,\sigma_{i-1}\,A_i - (\sigma_i + \sigma_{i-1})\,B_i + C_i$$

$$D_i' = \sigma_i\,\sigma_{i-1}\,B_i - (\sigma_i + \sigma_{i-1})\,C_i$$

$$E_i' = \sigma_i\,\sigma_{i-1}\,C_i$$

$$A_i = \frac{-64}{(\sigma_i - \sigma_{i-1})^4}\left[\frac{2}{3} f\left(\frac{\sigma_i + 3\sigma_{i-1}}{4}\right) - f\left(\frac{\sigma_i + \sigma_{i-1}}{2}\right) + \frac{2}{3} f\left(\frac{3\sigma_i + \sigma_{i-1}}{4}\right)\right]$$

$$B_i = \frac{64}{(\sigma_i - \sigma_{i-1})^4}\left[\left(\frac{5\sigma_i + 3\sigma_{i-1}}{6}\right) f\left(\frac{\sigma_i + 3\sigma_{i-1}}{4}\right)\right.$$

$$\left. - \left(\sigma_i + \sigma_{i-1}\right) f\left(\frac{\sigma_i + \sigma_{i-1}}{2}\right) + \left(\frac{3\sigma_i + 5\sigma_{i-1}}{6}\right) f\left(\frac{3\sigma_i + \sigma_{i-1}}{4}\right)\right]$$

$$C_i = \frac{-8}{(\sigma_i - \sigma_{i-1})^4} \quad \frac{2(\sigma_i + \sigma_{i-1})(3\sigma_i + \sigma_{i-1})}{3} f\left(\frac{\sigma_i + 3\sigma_{i-1}}{4}\right)$$

$$- \frac{(\sigma_i + 3\sigma_{i-1})(3\sigma_i + \sigma_{i-1})}{2} f\left(\frac{\sigma_i + \sigma_{i-1}}{2}\right)$$

$$+ \frac{2(\sigma_i + 3\sigma_{i-1})(\sigma_i + \sigma_{i-1})}{3} f\left(\frac{3\sigma_i + \sigma_{i-1}}{4}\right)$$

$$A_1' = A_1$$

$$B_1' = B_1 - \sigma_i A_1$$

$$C_1' = C_1 - \sigma_1 B_1$$

$$D_1' = D_1 - \sigma_1 C_1$$

$$E_1' = -\sigma_1 D_1$$

$$A_1 = \frac{1}{(a-b)}\left[\frac{(f_5 - f_7)}{(a-c)} - \frac{(f_6 - f_7)}{(b-c)}\right]$$

$$B_1 = \frac{1}{(a-b)}\left[\frac{(a+b+c)}{(b-c)}(f_6 - f_7) - \frac{(b+c+d)}{(a-c)}(f_5 - f_7)\right]$$

$$C_1 = \frac{(bc+bd+cd)}{(a-b)(a-c)} f_5 - \frac{(ac+ad+cd)}{(a-b)(b-c)} f_6 + \frac{(ab+ad+bd)}{(a-c)\,b-c)} f_7$$

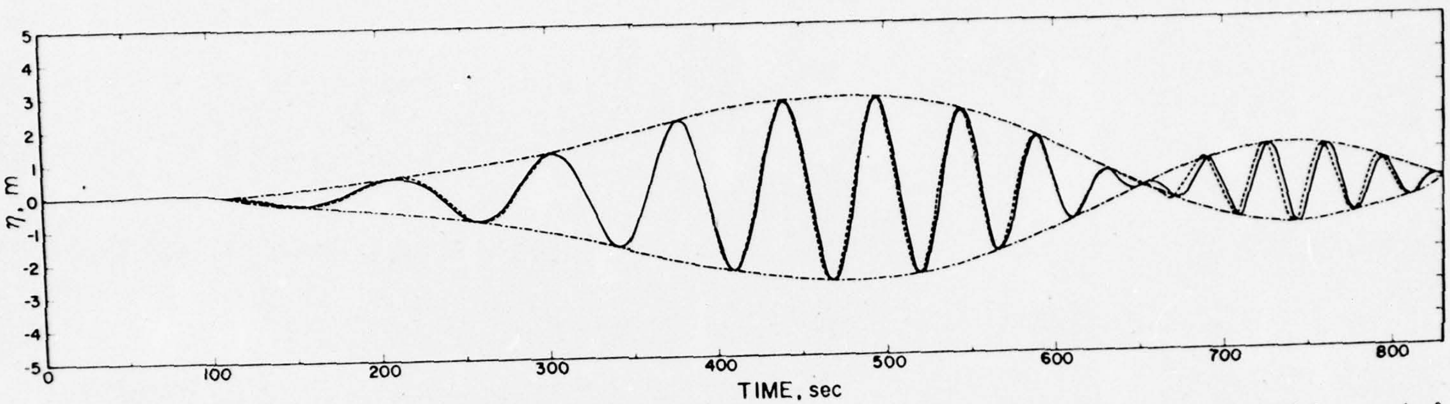

Fig. 1. Wave amplitude as a function of time 20 km from the source for a parabolic impulse. $R = 1.4$ km, $h = 5$ km, $I_0 = 5 \times 10^7$ dynes sec/cm^2. Method of stationary phase (solid line), integration method of this paper (dashed line), envelope (dot-dashed line).

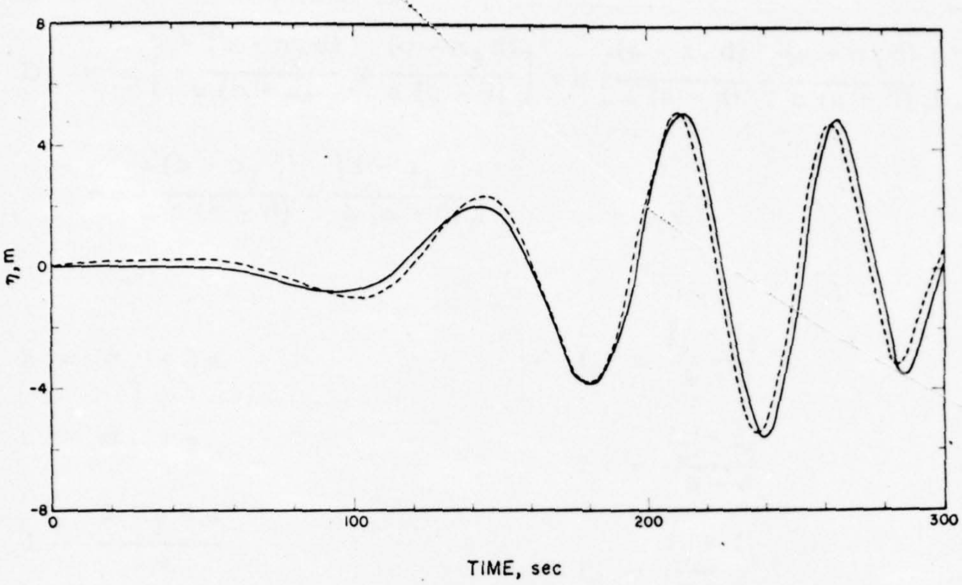

Fig. 2. Wave amplitude as a function of time 10 km from the source for a parabolic impulse. $R = 1.4$ km, $h = 5$ km, $I_0 = 5 \times 10^7$ dynes sec/cm^2. Method of stationary phase (solid line), integration method of this paper (dashed line).

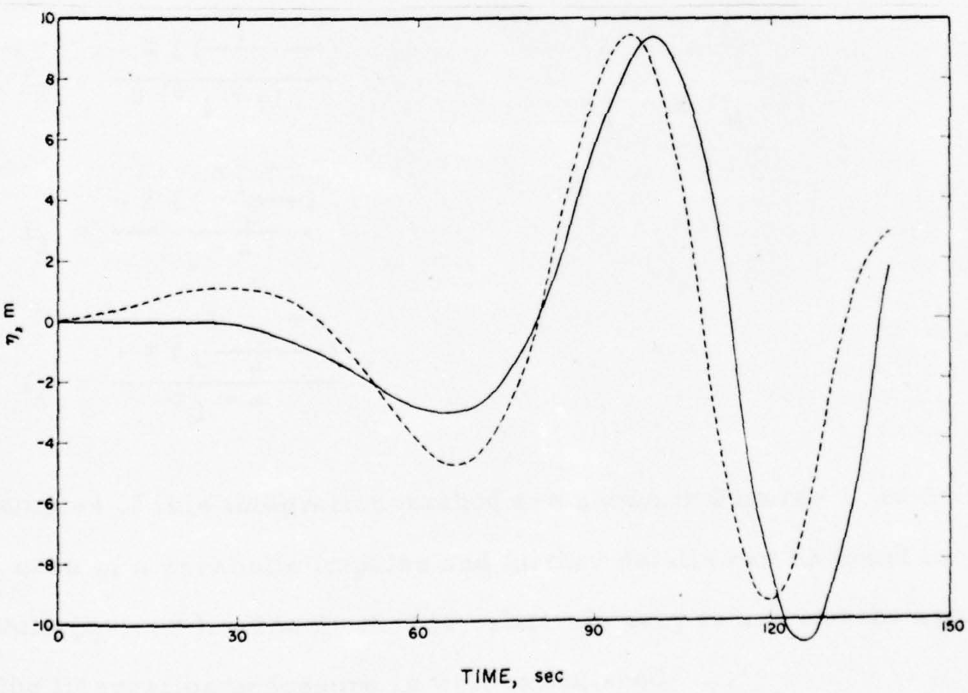

Fig. 3. Wave amplitude as a function of time 5 km from the source for a parabolic impulse. $R = 1.4$ km, $h = 5$ km, $I_0 = 5 \times 10^7$ dynes sec/cm^2. Method of stationary phase (solid line), integration method of this paper (dashed line).

$$D_1 = a\left[\frac{-(b - \alpha_2 d)}{b(b - d)} + \frac{(c - \alpha_3 d)}{c(c - d)}\right] + b\left[\frac{(a - \alpha_1 d)}{a(a - d)} - \frac{(c - \alpha_3 d)}{c(c - d)}\right]$$

$$+ c\left[\frac{(b - \alpha_2 d)}{b(b - d)} - \frac{(a - \alpha_1 d)}{a(a - d)}\right]$$

$$b = \sigma_1 + 3a$$

$$c = \sigma_1 + a$$

$$d = \frac{3\sigma_1 + a}{4}$$

$$f_1 = \frac{f(a)}{\sigma - a}$$

$$f_2 = \frac{-4f\left(\frac{\sigma_1 + 3a}{4}\right)}{3(\sigma_1 + a)}$$

$$f_3 = \frac{-2f\left(\frac{\sigma_1 + a}{2}\right)}{\sigma_1 - a}$$

$$f_4 = \frac{-4f\left(\frac{3\sigma_1 + a}{4}\right)}{\sigma_1 - a}$$

$$f_5 = \frac{f_1 - f_4}{a - d}$$

$$f_6 = \frac{f_2 - f_4}{b - d}$$

$$f_7 = \frac{f_3 - f_4}{c - d}$$

$$\alpha_1 = f_1/f_4$$

$$\alpha_2 = f_2/f_4$$

$$\alpha_3 = f_3/f_4$$

Examples of this integration method are shown in Figures 1 to 3 for the case of a parabolic impulse and further details can be found in Whalin (Ref. 60). The method is relatively easy to use and the accuracy of the integration procedure is well established.

2.5 Energy in Explosion Waves

One of the important considerations is the efficiency of an explosion as a wave producing mechanism. Many experimental tests have indicated that approximately 1-3% of the available energy appears in the form of surface waves for HE detonations. The percentage is considerably more uncertain in the case of nuclear explosions. This section gives the expressions for computing the potential energy of three surface deformations assumed for purposes of predicting the water waves. A derivation due to Sakurai is included for approximately relating this expression to field measurements of the envelope maximum at various stations, a distance r from GZ. Further theoretical expressions and curves are given showing the relationship between the potential and kinetic energy appearing in the wave train at any time t after the detonation.

2.5.1 Energy of the Surface Deformation

Letting $\eta_o(r_o)$ be the initial stationary surface deformation as in the preceding section, the potential energy is represented by

$$E = 2\pi \rho g \int_0^{\infty} \frac{\eta_o^2(r_o)}{2} r_o \, dr_o$$

In the case of a parabolic surface deformation represented by

$$\eta_o(r_o) = \begin{cases} \eta_o \left[1 - \left(\frac{r_o}{R}\right)^2\right] & r_o \le R \\ 0 & r_o > R \end{cases}$$

$$E = \pi \rho g \eta_o^2 \int_0^R \left[r_o - \frac{2 r_o^3}{R^2} + \frac{r_o^5}{R^4} \right] d r_o$$

$$= \pi \rho g \eta_o^2 \left[\frac{R^2}{2} - \frac{2 R^4}{4 R^2} + \frac{R^6}{6 R^4} \right]$$

$$= \frac{\pi \rho g \eta_o^2 R^2}{6}$$

The potential energy of the deformation with lip represented by

$$\eta_o (r_o) = \begin{cases} \eta_o \left[\frac{r_o^4}{3 R^4} - \frac{4 r_o^2}{3 R^2} + 1 \right] & r_o \le \sqrt{3}\, R \\ 0 & r > \sqrt{3}\, R \end{cases}$$

is given by

$$E = \pi \rho g \eta_o^2 \int_0^{\sqrt{3} R} \left[\frac{r_o^9}{9 R^8} - \frac{8 r_o^7}{9 R^6} + \frac{22 r_o^5}{9 R^4} - \frac{8 r_o^3}{3 R^2} + r_o \right] d r_o$$

$$= \pi \rho g \eta_o^2 \left[\frac{2 /7}{10} R^2 - 9 R^2 + 11 R^2 - 6 R^2 + \frac{3 R^2}{2} \right]$$

$$= \pi \rho g \eta_o^2 R^2 \left[\frac{42}{10} - 4 \right] = \frac{\pi \rho g \eta_o^2 R^2}{5}$$

A third case of interest is an initial deformation in the form of a parabolic cavity surrounded by a lip which is an extension of the cavity. This initail deformation is represented by

$$\eta_o (r_o) = \begin{cases} \eta_o \left\{ 1 - \frac{2 r_o^2}{R^2} \right\} & , \; r_o \le R \\ 0 & , \; r_o > R \end{cases}$$

and

$$E = \pi \rho g \eta_o^2 \int_o^R \left\{ 1 - \frac{4 r_o^2}{R^2} + \frac{4 r_o^4}{R^4} \right\} r_o \, d r_o$$

$$= \pi \rho g \eta_o^2 \left\{ \frac{R^2}{2} - \frac{4R^2}{4} + \frac{4R^2}{6} \right\}$$

$$= \frac{\pi \rho g \eta_o^2 R^2}{6}$$

It should be noted that the above surface deformation conserves the mass of the fluid domain, i.e., the amount of water above the undisturbed free surface is equal to the volume of the water cavity.

The above deformations are considered since they are commonly used in performing predictions of the surface waves generated by underwater explosions. Many other shapes and conditions could be used; however, the present state of the art has indicated that these initial conditions yield the most reliable prediction.

2.5.2 Kinetic and Potential Energy Distribution

At time zero, the mathematical models used for performing predictions assume that all the energy partitioned to the water is in the form of potential energy. One can study the partitioning between potential and kinetic energy as a function of time after detonation from the theoretical solution for the velocity potential. However, at large times after the explosion, the total energy is equally divided between kinetic and potential energy which is easily demonstrated as follows.

$$P.E. = \pi \rho g \int_0^A \sigma \bar{\eta}_o^{\,2}(\sigma) \cos^2 \{\Phi(\sigma)\, t\}\, d\sigma$$

where

$$\Phi(\sigma) = \sigma\, \varphi(\sigma) - \sqrt{\sigma \tanh \sigma}$$

$$P.E. = \frac{\pi \rho g}{2} (I_0 + I_1)$$

where

$$I_o = \int_0^A \sigma \bar{\eta}_o^{\,2}(\sigma)\, d\sigma$$

$$I_1 = \int_0^A \sigma \bar{\eta}_o^{\,2}(\sigma) \cos(2\Phi(\sigma)\, t)$$

$$I_o = \int_0^A \sigma \left\{ \int_0^\infty \eta_o(r_o)\, J_o(\sigma r_o)\, r_o\, d r_o \right\}^2 d\sigma$$

$$= \int_0^\infty d r_o \int_0^\infty d r_o'\, \eta_o(r_o)\, \eta_o(r_o')\, r_o r_o' \int_0^4 \sigma J_o(\sigma r_o')\, d\sigma$$

$$\lim_{A \to 0} \int_0^A \sigma\, J_o(\sigma r_o)\, J_o(\sigma r_o')\, d\sigma = \frac{1}{r_o'}\, \delta(r_o - r_o')$$

$$\therefore \; I_o = \int_0^\infty \eta_o^{\,2}(r_o)\, r_o\, d r_o = \frac{E}{\pi \rho g}$$

$$P.E. = \frac{E}{2}$$

A precise distribution of the potential and kinetic energy as a function of time can be evaluated from the expression for the velocity potential. The limiting value of an equal partitioning of the kinetic and potential energy will be reached relatively quickly.

2.5.3 Relationship Between the Wave Envelope and Initial Conditions

This section discusses an approximate method of estimating the explosive energy converted to water waves from field measurements of the wave amplitude as a function of time at any location r from SZ in water of constant depth. The method presented was first developed by Sakurai (Ref. 51) and greatly facilitates an estimate of wave energy.

The theoretical solution for the wave amplitude which yields very good agreement with field measurements provided the cavity parameters η_o and R are well chosen is given by

$$\eta(r,t) = \frac{2h\eta_o}{r} \frac{1}{\sigma^2} J_2\left(\frac{\sigma R}{h}\right) \sqrt{\frac{\sigma\varphi(\sigma)}{-\varphi'(\sigma)}} \cos\left(\sigma r - \sqrt{\sigma \tanh \sigma}\, t\right)$$ (parabolic cavity)

$$\eta(r,t) = \frac{4\eta_o h}{r\sigma^2} J_4\left(\sqrt{3}\,\frac{\sigma R}{h}\right) \sqrt{\frac{\sigma\varphi(\sigma)}{-\varphi'(\sigma)}} \cos\left(\sigma r - \sqrt{\sigma \tanh \sigma}\, t\right)$$ (crater with lip)

$$\eta(r,t) = \frac{\eta_o R}{r\sigma} J_3(\sigma R) \sqrt{\frac{\sigma\varphi(\sigma)}{-\varphi'(\sigma)}} \cos\left(\sigma r - \sqrt{\sigma \tanh \sigma}\, t\right)$$ (parabolic depression and cylindrical elevation)

It is very easy to perform measurements of the first envelope amplitude from wave records, which forms the basis of this method. The product of the first envelope amplitude and the distance r of the measurement station is given by

$$\eta_e r = 2h\eta_o \frac{1}{\sigma_m^2} J_2\left(\frac{\sigma_m R}{h}\right)\sqrt{\frac{\sigma_m \varphi(\sigma_m)}{-\varphi'(\sigma_m)}} \quad \text{[parabolic case]}$$

$$\eta_r r = 4h\eta_o \frac{1}{\sigma_m^2} J_4\left(\frac{\sqrt{3}\,\sigma_m R}{h}\right)\sqrt{\frac{\sigma_m \varphi(\sigma_m)}{-\varphi'(\sigma_m)}} \quad \text{[lip case]}$$

$$\eta_e r = \eta_o R \frac{1}{\sigma_m} J_3(\sigma_m R)\sqrt{\frac{\sigma_m \varphi(\sigma_m)}{-\varphi'(\sigma_m)}} \quad \text{(parabola and cylinder)}$$

where

σ_m = the value of σ at the envelope maximum

The validity of this method is subject to the following assumptions:

1. The measurement station is a large distance r from the explosion so that the above equations are valid.
2. The value of σ at the envelope maximum is sufficiently large so the asymptotic expression may be used for the functions $\varphi(\sigma)$ and $\varphi'(\sigma)$.
3. The wave train is represented relatively accurately by the above equations provided proper deformation parameters η_o and R are chosen.

Parabolic: $$\eta_e r = 2\sqrt{2}\ \eta_o R \ \frac{J_2\left(\frac{\sigma_m R}{h}\right)}{\left(\frac{\sigma_m R}{h}\right)}$$

Crater with Lip: $$\eta_e r = 4\sqrt{6}\ \eta_o R \ \frac{J_4 \frac{\sqrt{3}\ \sigma_m R}{h}}{\left(\frac{\sqrt{3}\ \sigma_m R}{h}\right)}$$

Cylinder and Parabola: $\eta_e r = \sqrt{2}\ \eta_o R \ J_3 (\sigma_m R)$

Parabolic case: The first maximimum of $\eta_e r$ is given by the first maximum of $J_2(x)/x$, given by

$$\frac{\sigma_m R}{h} = 2.2999$$

$$\frac{J_2\left(\frac{\sigma_m R}{h}\right)}{\left(\frac{\sigma_m R}{h}\right)} = 0.17996$$

Therefore

$$\eta_e r \approx 0.509\ \eta_o R$$

$$\eta_o R \approx 1.964\ \eta_e R$$

From the previous section, the potential energy of a parabolic cavity is given by

$$E = \frac{\pi \rho g \eta_o^2 R^2}{6}$$

Upon substitution

$$E \approx \frac{3.86 \pi \rho g}{6} \eta_{max}^2 r^2$$

$$E \approx 129 \eta_{max}^2 r^2 \text{ ft. lbs.}$$

Lip Case:

In this case the maximum of $J_4(x)/x$ is 0.0787, resulting in

$$\eta_{max} r = 0.771 \eta_o R$$

$$\eta_o R = 1.299 \eta_{max} r$$

The total energy of the initial deformation is given by

$$E \approx 66.2 \eta_{max}^2 r^2 \text{ ft. lbs.}$$

Cylinder and Parabola Case:

The maximum of the wave envelope is merely at the maximum of $J_3(x)$ which gives

$$\eta_{max} r = 0.614 \eta_o R$$

$$\eta_o R = 1.63 \eta_{max} r$$

$$E \approx \frac{\pi \rho g \eta_o R^2}{6}$$

$$E \approx 88.8\, \eta_{max}^2\, r^2$$

2.6 Initial Conditions

It has been demonstrated in the previous section that one has a wide variety of initial conditions in order to simulate the wave generation mechanism for an explosion. Intuitively, for the case of an air explosion, a time dependent pressure, $P(r_o, \tau)$, would seem to be the logical choice of an initial condition. Unfortunately, to the knowledge of the authors, there have been no theoretical developments other than those presented in the previous section. It has been demonstrated (communicated through private discussion with Drs. Van Dorn and Olsen) that the integration of pressure measurements from field tests to give an initial impulse and the subsequent use of an initial impulse for predicting waves results in an erroneous prediction by more than an order of magnitude. The use of a time dependent pressure is the next logical step toward a realistic model and would comprise both an interesting and valuable research task. However, in this report we are interested in underwater explosions, primarily because the energy converted to water waves is significantly larger than for air explosions. It has been well established that the use of an impulse or a surface deformation as an initial condition results in a wave train which is qualitatively similar to those measured from explosions. After considerable trial and error, it was further demonstrated that a parabolic deformation or impulse of the form

$$\eta_o(r_o) = \begin{cases} \eta_o\left[1 - \left(\frac{r_o}{R}\right)^2\right] & r_o \le R \\ 0 & r_o > R \end{cases}$$

$$I_o(r_o) = \begin{cases} I_o\left[1 - \left(\frac{r_o}{R}\right)^2\right] & r_o \le R \\ 0 & r_o > R \end{cases}$$

resulted in the best comparison with experiment provided the two parameters η_o, R or I_o, R were properly selected. The only objection to this form of initial condition is the fact that an initial deformation consisting of a net addition or removal of fluid from the domain results in the prediction of a discontinuity in the water surface at the front of the wave train. Therefore any prediction of the leading wave will be in error, and under certain conditions, the leading wave is of considerable interest due to its long period and large shoaling coefficient.

Whalin (Ref. 57) developed a model consisting of a water crater with a lip where the volume of fluid in the domain is conserved, thus eliminating the discontinuity at the front of the wave train. This deformation is given by

$$\eta_o (r_o) = \begin{cases} \eta_o \left[\dfrac{r^4}{3R^4} - \dfrac{4r^2}{3R^2} + 1 \right] & r \le \sqrt{3}\,R \\ 0 & r > \sqrt{3}\,R \end{cases}$$

and is illustrated in the following figure.

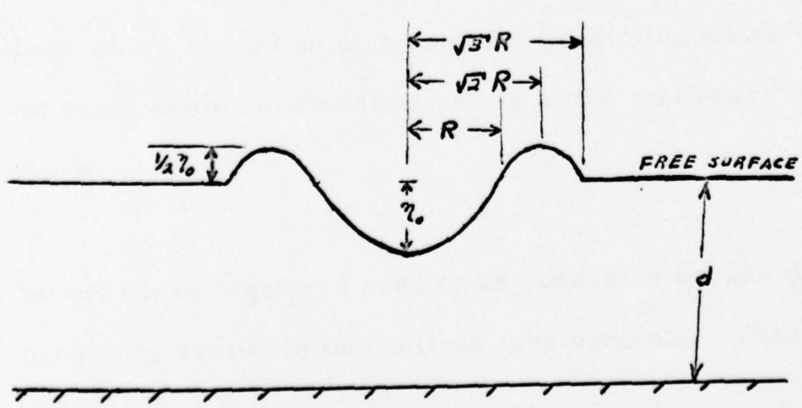

Figure 4 Initial Deformation with a Lip

Recent preliminary investigations indicate that perhaps the combination of a parabolic deformation and cylindrical elevation gives a better prediction of the entire wave train than either of the previous two initial conditions. This deformation is given by

$$\eta_o(r_o) = \eta_o \left[= 1 - \frac{2r^2}{R^2} \right] \quad r \leq R$$

$$0, \quad r > R$$

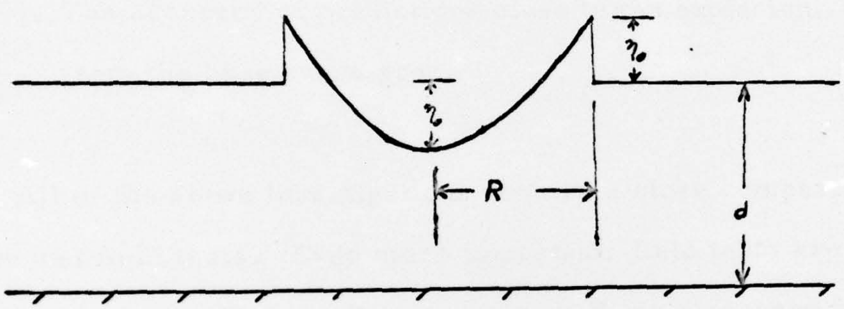

Figure 5 Cylindrical Elevation plus Parabolic Depression

Therefore, through a considerable amount of investigation, the initial conditions which are commonly used for predicting water waves generated by underwater explosions consist of one of the stationary surface deformations above.

Several investigations need to be conducted to take full advantage of the presently available theoretical developments. These include:

1. The accuracy of the above initial conditions for predicting wave amplitudes near the front and end of the wave envelope. It is well established that periods and maximum amplitudes can be predicted accurately.
2. The applicability of time dependent deformations or an initial velocity field for developing a better model of the initial conditions.
3. The accuracy of predictions close to the explosion, yet away from the base surge area.

All of the above investigations require a close comparison between theory and field tests. Even more important, field tests are a necessity in evaluating the initial conditions η_o and R as discussed in the following section.

2.7 The Role of Field Tests in Theoretical Developments

The previous theoretical developments are all based on the linear theory of surface waves in water of constant depth. Obviously, the hydrodynamic phenomena in the immediate vicinity of an explosion constitute a highly nonlinear problem. Initially, it seems absurd to expect a linear theory assuming a mere deformation of the water surface to yield reliable predictions of the water waves produced. However, through comparisons of the theoretical results with field tests, it was well established that the linear theory produced qualitatively satisfactory results. The only problem remaining was to relate the parameters η_o and R of the initial deformation to charge weight, W, submergence depth, z, and water depth, h. In the case of explosions in deep water, defined as water sufficiently deep such that the bottom has a negligible effect on the generation of the surface waves, the only parameters of consequence are W and z.

The present state of the art, which seems to be quite satisfactory for logistic purposes, could not have been achieved without the performance of this particular experimental test series. The range of charge weights and submergence depths used in this test series was sufficient to result in a reliable empirical evaluation of the parameters $\eta_o(z, W)$ and $R(z, W)$. This initial evaluation of these parameters was performed by Whalin (Ref. 57) without all the data from this test series. These parameters are reevaluated (Ref. 51) in Section 4 of this report with the inclusion of all data from the program. The method of evaluation is presented in the following paragraphs.

A dimensional analysis of the problem was attempted, however, this procedure failed due to the lack of any consistent plot of the data in terms of dimensionless parameters. This was probably due to

many factors: the fact that gravity could not be scaled, a change in efficiency of detonation with increasing charge weight, the speed of detonation could not be scaled, etc.

Approaching the problem from an empirical point of view, the following steps were undertaken:

1. A confirmation of the $1/r$ amplitude decay of the envelope with distance. This has been verified (Refs. 57, 50, 51).
2. An empirical formula for the period, wave number, and wave length associated with the envelope maximum (all are related to each other) as a function of z and W.
3. An empirical relationship for the product of the envelope amplitude and distance from SZ ($\eta_{max}\, r$ which was shown constant by 1. above) as a function of z and W for any one detonation.

The relationships above can be represented by

$$k_{max} = f_1\,(z, W)$$

$$\eta_{max}\, r = f_2\,(z, W)$$

In order to evaluate η_o and R from these relationships, one must refer to the asymptotic solution for the case of an initial deformation

$$\eta\,(r, t) = \frac{\overline{\eta_o}(\sigma)}{r} \sqrt{\frac{\sigma\,\varphi(\sigma)}{-\varphi'(\sigma)}} \cos\,(\sigma r - \sqrt{\sigma \tanh \sigma}\; t)$$

where the envelope is represented by

$$\eta_e\,(r, t) = \frac{\overline{\eta_o}(\sigma)}{r} \sqrt{\frac{\sigma\,\varphi(\sigma)}{-\varphi'(\sigma)}}$$

Note that $\overline{\eta_o}(\sigma)$ contains both parameters η_o and R and

$$\overline{\eta_o}(\sigma) = \frac{2\eta_o}{\sigma^2} J_2(\sigma R) \quad \text{parabolic deformation}$$

$$\overline{\eta_o}(\sigma) = \frac{4\eta_o}{\sigma^2} J_4(\sqrt{3}\,\sigma R) \quad \text{deformation with a lip}$$

$$\overline{\eta_o}(\sigma) = \frac{\eta_o R}{\sigma^2} J_3(\sigma R) \quad \text{cylinder and parabola}$$

We now proceed with the evaluation of R. Noting that $\sigma = kh$ or $\sigma_{max} = k_{max} h = h f_1(z, W)$, the value of σ at the envelope maximum, σ_{max}, is the solution of $\frac{d(\eta_e r)}{dm} = 0$. Therefore, R is the solution of

$$\left[\frac{d(\eta_e r)}{d\sigma}\right]_{\sigma = \sigma_{max} = h f_1(z, W)} = 0,$$ shown below for the specific cases above.

$$\left[\frac{d}{d\sigma}\left\{\frac{1}{\sigma^2} J_2(\sigma R)\sqrt{\frac{\sigma\varphi(\sigma)}{-\varphi'(\sigma)}}\right\}\right]_{\sigma = h f_1(z, W)} = 0 \quad \text{parabolic case}$$

and

$$\left[\frac{d}{d\sigma}\left\{\frac{1}{\sigma^2} J_4(\sqrt{3}\,\sigma R)\sqrt{\frac{\sigma\varphi(\sigma)}{-\varphi'(\sigma)}}\right\}\right]_{\sigma = h f_1(z, W)} = \quad \text{lip case}$$

$$\left[\frac{d}{d\sigma}\left\{\frac{1}{\sigma} J_3(\sigma R)\sqrt{\frac{\sigma\varphi(\sigma)}{-\varphi'(\sigma)}}\right\}\right]_{\sigma = h f_1(z, W)} = 0 \quad \text{parabola and cylinder}$$

The solution for η_o is now a simple matter, upon substitution

$$\eta_o = \frac{f_2(z, W)}{2} \left[\frac{\sigma^2}{J_2(\sigma R)} \sqrt{\frac{-\varphi'(\sigma)}{\sigma\varphi(\sigma)}} \right]_{\sigma = h f_1(z, W)} \quad \text{parabolic case}$$

$$\eta_o = \frac{f_2(z, W)}{4} \left[\frac{\sigma^2}{J_4(\sqrt{3}\,\sigma R)} \sqrt{\frac{-\varphi'(\sigma)}{\sigma\varphi(\sigma)}} \right]_{\sigma = h f_1(z, W)} \quad \text{lip case}$$

$$\eta_o = f_2(z, W) \left[\frac{\sigma}{R J_3(\sigma R)} \sqrt{\frac{-\varphi'(\sigma)}{\sigma\varphi(\sigma)}} \right]_{\sigma = h f_1(z, W)} \quad \text{cylinder and parabola}$$

The functions f_1 and f_2 are evaluated in Section 3 of this report along with a table of η_o and R as a function of z and W .

One must realize at this point that the values of η_o and R obtained in this manner will not necessarily have any resemblance to the actual water crater formed by the explosion. A theoretical solution has been developed which adequately describes the qualitative features of the wave train. The experimental data are then used to evaluate the initial conditions of this model as a function of z and W so that the predictions will result in empirically derived values of the wave number and amplitude at the envelope maximum.

It is readily apparent that reliable predictions could not be performed without both the theoretical analysis of problem and the performance of well planned and executed field tests.

REFERENCES

1. Cauchy, A. L., Proc. Lond. Math. Soc. (2) i 473.

2. Poisson, S., "Memoire sur la theorie des ondes," Mem de l'Acad. Roy. des Sciences, i (1816).

3. Burkhardt, H., "Entwickleungen noch oscillirenden Functionen," Leikzig, 1901, pp. 429, 439.

4. Lamb, Horace, "On Deep Water Waves," Proc. Lond. Math. Soc., Nov. 10, 1904, pp. 371-400.

5. Rayleigh, J. W., Scientific Papers, Vol. III, p. 129.

6. Kelvin, W. T., Phil. Mag., Vol. 13, p. 11 (1907)

7. Watson, G. N., "Theory of Bessel Functions."

8. Burnside, W., "On Deep Water Waves Resulting from a Limited Original Disturbance," Proc. Lond. Math. Soc. (1) xx. 22 (1888).

9. Dawson, H. G., Proc. Lond. Math. Soc., (1) xxix 519 (1898).

10. Terazawa, K., Science Reports of the Univ. of Tokyo, Vi. 171 (1917)

11. Pidduck, F. B., "On the Propagation of a Disturbance in a Fluid under Gravity," Proc. Roy. Soc. A, lxxxii 347 (1910).

12. Thompson, Sir W., "On the Waves Produced by a Single Impulse in Water of any Depth or in a Dispersive Medium," Proc. Roy. Soc., xlii 80 (1887).

13. Pidduck, F. B. M."The wave Problem of Cauchy and Poisson for Finite Depth and Slightly Compressible Fluid," Proc. Roy. Soc. lxxxvi 396 (1912).

14. Stokes, G. G., "On the Numverical Calculation of a Class of Definite Integrals and Infinite Series," Camb. Trans. ix (1850).

15. Havelock, T. H., "The Propagation of Waves in Dispersive Media," Proc. Roy. Soc., ixxxi 398 (1908).

16. Hasegawa, K. and Sono, K., "Waves Produced by Sudden Depression of the Bottom in the Immediate Neighborhood of a Point," Bull. Cent. Met. Obs. Japan, Vol. II, 1915.

17. Sono, K., "On the Waves Caused by a Sudden Deformation of a Finite Portion of the Bottom of Sea of Uniform Depth," Geophys. Mag., 10, pp. 21-41, 1936.

18. Terazawa, K., "On the Oscillations of the Deep-Sea Surface Caused by a Local Disturbance," Sci. Rep. Tokoku Univ., 6, pp. 169-181, 1917.

19. Nomura, Y., "On the Waves in Water of Finite Depth due to Disturbance on the Surface," Sci. Rep. Tokoku Univ., 26, pp. 687-713, 1937.

20. Segawa, K. and Konai, K., "On the Transmission of Tsunamis in a Sea of any Depth," Bull. Earthquake Research Inst., 20, pp. 254-264 1942.

21. Takahashi, R., "On Seismis Sea Waves Caused by Deformations of the Sea Bottom," Bull. Earthquake Research Inst., 20, pp. 375-400, 1945.

22. Ibid., Second Report, 23, 23-35, 1945.

23. Ibid., Third Report, "The One-Dimensional Source," 25, 5-8, 1947.

24. Ichiye, T., "On the Theory of Tsunami," Oceanographical Magazine, 2, 83-100.

25. Honda, H. and Nakamura, K., "‌ e Waves Caused by One-Dimensional Deformation of the Bottom Shallow Sea of Uniform Depth," Sci. Rep., Tokoku Univ., Ser. 5, Geophysics, 3, pp. 133-137, 1951.

26. Nakamura, K., "On the Waves Caused by the Deformation of the Bottom of the Sea, I," Sci. Rep., Tokoku Univ., Geophysics, pp. 167-176, 1953.

27. Unoki, S., and Nakano, M., "On the Cauchy-Poisson Waves Caused by the Eruption of a Submarine Volcano", (first paper) Oceano. Mag. 4 (4), pp. 119-141 (1951-1953).

28. Ibid., Second paper, 5 (1), pp. 1-13 (1951-1953).

29. Ibid., Third paper, 4 (3-4), pp. 139-150 (1951-1953).

30. Kajiura, K. . "The Leading Wave of a Tsunami," Bull. Earthquake Res. Institute, Tokyo Univ., 41, pp. 535-571 (1963).

31. "Underwater Explosion Research," Vol. I, The Gas Globe, Office of Naval Research, Department of the Navy.

32. Ibid., Vol. II

33. Cole, R.H., "Underwater Explosions," Princeton University Press, 1948.

34. Snay, H.G. and E.A. Christian, "Underwater Explosion Phenomena: The Parameters of a Non-Migrating Bubble Oscillating in an Incompressible Medium," NAVORD Report 2437, 1952.

35. Snay, H.G., "Migration of Explosion Bubbles in a Rotating Test Tank," NOLTR, 61-145, 1962.

36. Snay, H.G., "Underwater Explosion Phenomena: The Parameters of Migrating Bubbles," NAVORD Report 4185, 1962.

37. Stoker, J.J., "Water Waves," Interscience Publishers Inc., New York, 1957.

38. Kranzer, H.C. and Keller, J.B., "Water Waves Produced by Explosions," IMM-NYU 222, September 1955.

39. Prins, J.E., "Characteristics of Waves Generated by a Local Surface Disturbance," Wave Research Laboratory of Engineering Research, Univ. of Calif., Berkeley, Series 99, Issue 1, August 1956.

40. Kaplan, K. and Wallace, N.R., "Surface Waves (From Nuclear Explosions," United Research Services Report 190-1, March 1962, DASA-1240), Secret.

41. Kaplan, K. and Wallace, N.R., "A Study of Explosion-Generated Surface Water Waves," Broadview Research Progress Report 162-2, February 1961, AD 348-228, Secret.

42. Kaplan, K. and Wallace, N.R., "A Study of Explosion-Generated Surface Water Waves," Broadview Research Progress Report BRC 162-1, August 1960, AD 349-227, Confidential.

43. Ibid., Progress Report 162-3, August 1961, AD 349-229, Confidential.

44. Kaplan, K., "A Study of Explosion-Generated Surface Water Waves," United Research Services Progress Report 162-4, February 1962, AD 349-216, Confidential.

45. Kaplan, K. and Wilton, C., "A Study of Explosion-Generated Surface Waves," United Research Services Progress Report B162-5, May 1962, AD 349-218, Confidential.

46. Bertram, J.R. and Kaplan, K., "A Study of Explosion-Generated Surface Water Waves," United Research Services Progress Report B162-6, September 1962, AD 349-217, Confidential.

47. Kaplan, K., Goodale, T.C., and Wallace, N.R., "A Study of Explosion-Generated Surface Water Waves," United Research Services Progress Report 162-7, March 1963, AD 349-215, Secret.

48. Kaplan, K. and Criner, O.C., "A Study of Explosion-Generated Surface Water Waves," United Research Services Final Report 162-8, December 1963, Secret.

49-
52. WES Reports on DASA Wave Program.

53. Whalin, R.W. and Kent, R.E., "Water Waves Produced by Impulsive Energy Sources. Part III: Data Analysis and Scaling Relations," NMC-ONR 62, October 1962, Secret.

54. Whalin, R.W. "Ibid. Part V: Propagation," NMC-ONR 62, September 1963.

55. Whalin, R.W. and Kent, R.E., "Ibid. Part VI: Data Analysis," December 1963, Secret.

56. Whalin, R.W., "Research on the Generation and Propagation of Water Waves Produced by Underwater Explosions. Part I: Propagation in the Area Near the Source," December 1964, Confidential.

57. Whalin, R. W., "Ibid. Part II: A Prediction Method," January 1965, Secret.

58. Whalin, R. W., "Ibid. Part IV: A Generalized Method for Predicting the Surface Water Wave Spectrum," Vol. I, Final Report, NMC-ONR 64, July 1965, Secret.

59. Whalin, R. W., "Ibid. Vol. II," July 1965 .

60. Whalin, R. W., "Water Waves Produced by Underwater Explosions: Propagation Theory for Regions near the Explosion," Journal of Geophysical Research, Vol. 70, No. 22, November 15, 1965.

61. Van Dorn, W. G., "Theoretical and Experimental Study of Impulsively Generated Waves (U)," Final Report, Nonr 233(35), University of California, Scripps Institution of Oceanography, 1 April 1964, Secret.

62. Van Dorn, W. G., "Operation Redwing -- Project 1.96, Indirect Water Waves from Large Yield Bursts (U)," WT-1308, H. F. C., DASA Scandia Base, Albuquerque, N. M., 9 June 1959, Secret.

63. Van Dorn, W. G., "Water Wave Measurements During Operation Dominic," WT-2058, Final Report, Task Group 8.5.2, AEC (11-1), GEN-10 15 May 1964, Secret.

64. Van Dorn, W. G., "Water Waves from 10,000 lb. High-Explosive Charges," Final Report, Operation Hydra II-A, Contract Nonr 2216 (15) University of California, Scripps Institution of Oceanography, La Jolla, Calif., SIO 63-20.

65. Le Méhauté, B. and Whalin, R. W., "Contribution to the Mono Lake Experiments. Vol. I: Predictions of the Water Waves and Run-up from 10,000 lb TNT Explosions," NESCO Report S-256-2, October 1965.

66. Whalin, R. W., "Ibid. Vol. II, Final Report, The Linear Theory of Water Waves Generated by Explosions," NESCO Report S-256-2, October 1965.

67. Le Méhauté, B.; Whalin, R. ; Koh, R.; Hwang, L. S.; Webb, and Divoky, D., "Hydrodynamic Effects of Nuclear Explosions. Vol. I: State of the Art," NESCO Report S-244-1, May 1966.

68. Le Mehaute, B.; Whalin, R.; and Divoky, D., "Ibid Vol. III: Coastal Inundation at Los Angeles, California due to Offshore Nuclear Bursts," NESCO Report S-244-3, May 1966, Confidential.

69. Le Méhauté, B.; Whalin, R.; and Divoky, D., "Ibid. Vol. IV: Five City Study," NESCO Report S-244-4, June 1966, Confidential.